Cosmological models described by a mixture of van der Waals fluid and dark energy

G. M. Kremer*

Departamento de Física, Universidade Federal do Paraná, Caixa Postal 19044, 81531-990 Curitiba, Brazil

The Universe is modeled as a binary mixture whose constituents are described by a van der Waals fluid and by a dark energy density. The dark energy density is considered either as the quintessence or as the Chaplygin gas. The irreversible processes concerning the energy transfer between the van der Waals fluid and the gravitational field are taken into account. This model can simulate: (a) an inflationary period where the acceleration grows exponentially and the van der Waals fluid behaves like an inflaton; (b) an inflationary period where the acceleration is positive but it decreases and tends to zero whereas the energy density of the van der Waals fluid decays; (c) a decelerated period which corresponds to a matter dominated period with a non-negative pressure; and (d) a present accelerated period where the dark energy density outweighs the energy density of the van der Waals fluid.

I. INTRODUCTION

One of the main objectives of the cosmological models is the description of the different phases of the Universe concerning the time evolution of its acceleration field. The first epoch represents a rapid expansion of the Universe which is known as the inflationary period. Most of the theories that describe this period makes use of a scalar field which is related to a hypothetical particle, the so-called inflaton (see e.g. the works [1] and the references therein). The second period refers to a past decelerated epoch where the energy densities of the radiation and matter fields outweigh the scalar field. The present epoch is characterized by an accelerated Universe dominated by a dark non-baryonic matter and a dark energy density. The most common models for the dark energy density make use of the equations of state of the quintessence [2] and of the Chaplygin gas [3].

Another model for the Universe was proposed recently by Capozziello and co-workers [4] who used the equation of state of a van der Waals fluid in order to analyze the accelerated behavior of the Universe. The advantage of this model is that it can describe the transition from a scalar field dominated epoch to a matter field dominated period without introducing scalar fields.

The objective of the present work is to investigate a Universe described by the van der Waals equation of state. In section 2 we follow the works [4] and model the Universe as a non-dissipative one-component van der Waals fluid. We determine the time evolution of the acceleration, energy density and pressure fields and show that this model can simulate the two phases of the Universe beginning with an accelerated period and going into a decelerated epoch. Although this model could describe the transition from an inflationary period to a matter dominated epoch it could not simulate the present accelerated period of the Universe. For this end we have

modeled in section 3 the Universe as a binary mixture of a van der Waals fluid and dark energy. The dark energy is also regarded either as the quintessence or as the Chaplygin gas. Furthermore, we have consider the irreversible processes related to the energy transfer between the van der Waals fluid and the gravitational field, since it is very questionable to get rid of the dissipative effects during the evolution of the Universe. Among other results, it was shown that this model can simulate: (a) an inflationary period where the acceleration grows exponentially and the van der Waals fluid behaves like an inflaton; (b) an inflationary period where the acceleration is positive but it decreases and tends to zero whereas the energy density of the van der Waals fluid decays; (c) a decelerated period which corresponds to a matter dominated period with a non-negative pressure; and (d) a present accelerated period where the dark energy density outweighs the energy density of the van der Waals fluid. Units have been chosen so that $c = 1$ and $8\pi G/3 = 1$.

II. UNIVERSE AS A VAN DER WAALS FLUID

In this section we consider a model for a spatially flat, homogeneous and isotropic Universe where the energy-momentum tensor of the sources of the gravitational field is described by a perfect fluid with a van der Waals equation of state in the absence of dissipative processes. The energy-momentum tensor $T^{\mu\nu}$ of the sources is given by

$$T^{\mu\nu} = (\rho_w + p_w)U^\mu U^\nu - p_w g^{\mu\nu}, \qquad (1)$$

where U^μ (such that $U^\mu U_\mu = 1$) is the four-velocity. The pressure of the van der Waals fluid p_w is related to its energy density ρ_w by (see, e.g. [5])

$$p_w = \frac{8w_w \rho_w}{3 - \rho_w} - 3\rho_w^2. \qquad (2)$$

In the above equation the pressure p_w and the energy density ρ_w are written in terms of dimensionless reduced variables and w_w is a parameter connected with a reduced
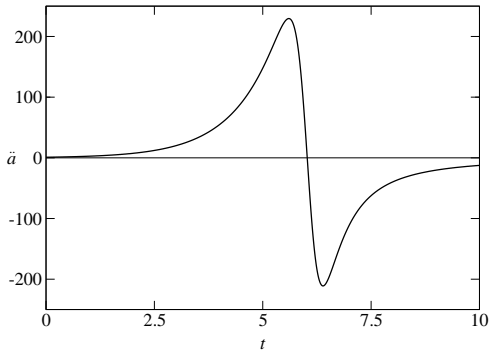
*Electronic address: kremer@fisica.ufpr.br

FIG. 1: Acceleration vs time for Universe as a van der Waals fluid.

FIG. 2: Energy density (straight line) and pressure (dashed line) vs time for Universe as a van der Waals fluid.

temperature. Here w_w will be identified with the coefficient of the barotropic formula, since for small values of ρ_w we have $p_w \propto w_w \rho_w$.

By considering the Robertson-Walker metric and a co-moving frame one can get from the conservation law $T^{\mu\nu}{}_{;\nu} = 0$ for the energy-momentum tensor the following balance equation for the energy density

$$\dot{\rho}_w + 3H(\rho_w + p_w) = 0. \tag{3}$$

In the above equation $H = \dot{a}(t)/a(t)$ denotes the Hubble parameter, $a(t)$ is the cosmic scale factor and the over-dot refers to a differentiation with respect to a dimensionless time t.

The connection between the cosmic scale factor and the energy density is given by the Friedmann equation

$$H^2 = \rho_w, \tag{4}$$

whose differentiation with respect to time leads to the time evolution of the cosmic scale factor

$$\dot{H} + \frac{3}{2}\left[H^2 + \frac{8w_w H^2}{3 - H^2} - 3H^4\right] = 0. \tag{5}$$

The solution of the second-order differential equation (5) for $a(t)$ can be found by specifying initial values for $a(t)$ and $H(t)$ at $t = 0$, for a given value of the parameter w_w. Here we have chosen the values $a(0) = 1$ for the cosmic scale factor and $H(0) = 1$ for the Hubble parameter (by adjusting clocks) and in the following we shall comment on the possible values for the parameter w_w.

In the interval $0 \leq w_w < 1/2$ the energy density grows with time, the pressure is always negative whereas the acceleration is always positive. One can conclude that this solution does not model properly the evolution of the Universe.

The case where $w_w = 1/2$ is the most interesting and in figures 1 and 2 we have plotted the acceleration, the energy density and the pressure as functions of the time. One can infer from figures 1 and 2 that the time interval between $0 \leq t \lesssim 5.6$ refers to an inflationary period since the acceleration grows exponentially and the van
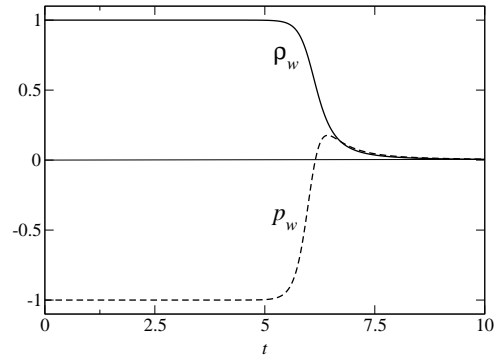
der Waals fluid behaves like an inflaton with an equation of state $p_w = -\rho_w$. The time interval between $5.6 \lesssim t \lesssim 6$ is also another inflationary period since the acceleration is positive but it decreases and tends to zero whereas the energy density decays and the pressure is always negative. For $6 \lesssim t \lesssim 6.4$ the acceleration is negative and attains its minimum value whereas the pressure is positive and reaches its maximum value, hence this time interval can be interpreted as a matter dominated period. The next time interval between $6.4 \lesssim t < \infty$ refers also to a matter dominated period since the acceleration is always negative and the energy density and the pressure decay and tend to zero which corresponds to a dust dominated period.

For $1/2 < w_w < 2/3$ the energy density decays with time, the pressure grows from a negative value to a maximum positive value and decays at large values of time while there exists a period of acceleration followed by a period of deceleration. This interval for w_w could simulate an inflationary period ($p_w < 0$), where the energy density of the inflaton decays, followed by a matter ($p_w \neq 0$) and dust ($p_w \rightarrow 0$) dominated periods.

The energy density and the pressure have the same behavior as the former interval for $2/3 \leq w_w < 3/4$ but there exists only a period of deceleration and one can infer that this solution does not model properly the evolution of the Universe.

For $w_w \geq 3/4$ the pressure is always positive whereas the acceleration is always negative and one can conclude that it refers only to a matter ($p_w \neq 0$) and dust ($p_w \rightarrow 0$) dominated periods.

Although the van der Waals fluid could describe the transition from an inflationary period to a matter dominated epoch it could not simulate the present accelerated period of the Universe. Moreover, in the present model we have not consider the irreversible processes concerning the energy transfer between the van der Waals fluid and the gravitational field. This will be the subject of the next section.

III. UNIVERSE AS A BINARY MIXTURE OF VAN DER WAALS FLUID AND DARK ENERGY

If dissipative effects are taken into account and we consider the Universe as a binary mixture of a van der Waals fluid with dark energy, the energy-momentum tensor reads

$$T^{\mu\nu} = (\rho_w + \rho_d + p_w + p_d + \varpi)U^\mu U^\nu - (p_w + p_d + \varpi)g^{\mu\nu}. \quad (6)$$

Above ρ_d is the dark energy density, p_d its pressure while ϖ denotes the non-equilibrium pressure which is connected with the irreversible processes (see e.g. [6, 7]).

The evolution equation for the energy density of the mixture is now given by

$$\dot{\rho}_w + \dot{\rho}_d + 3H(\rho_w + \rho_d + p_w + p_d + \varpi) = 0, \quad (7)$$

while the Friedmann equation becomes

$$H^2 = \rho_w + \rho_d. \quad (8)$$

The dark energy is supposed to interact only with itself and it is minimally coupled to the gravitational field, so that its energy density balance equation decouples from that of the van der Waals fluid, and we obtain from (7) two balance equations

$$\dot{\rho}_d + 3H(\rho_d + p_d) = 0, \qquad \dot{\rho}_w + 3H(\rho_w + p_w) = -3H\varpi. \quad (9)$$

The term $-3H\varpi$ on the right-hand side of $(9)_2$ can be interpreted as the energy density production rate of the van der Waals fluid. It is connected with the energy density production rate of the gravitational field by (see e.g. [6])

$$\dot{\rho}_G + 3H(\rho_G - p_w - p_d) = 3H\varpi, \quad (10)$$

where ρ_G is the energy density of the gravitational field. Hence, the non-equilibrium pressure is responsible for the irreversible transfer of the energy density of the gravitational field to the van der Waals fluid.

Two models for the dark energy are considered here, namely the quintessence ($\rho_d = \rho_X$) and the Chaplygin gas ($\rho_d = \rho_c$). The equations of state for these two models are given by

$$\begin{cases} p_X = w_X \rho_X, & \text{with } w_X < -1/3, \\ p_c = -A/\rho_c, & \text{with } A = \text{constant} > 0. \end{cases} \quad (11)$$

For the motivation of the equations of state above one is referred to e.g. [2, 3] and the references therein.

The equations of state (11) permit us to get from $(9)_1$ a relationship between the energy density of the quintessence and the cosmic scale factor as well as the corresponding one for the Chaplygin gas. In terms of dimensionless quantities these relationships read

$$\frac{\rho_X}{\rho_w^0} = \frac{\rho_X^0}{\rho_w^0}\left(\frac{1}{a}\right)^{3(w_X+1)}, \qquad \frac{\rho_c}{\rho_w^0} = \frac{\rho_c^0}{\rho_w^0}\frac{1}{\sqrt{1+\psi}}\sqrt{1+\frac{\psi}{a^6}}, \quad (12)$$

where ρ_X^0/ρ_w^0 and ρ_c^0/ρ_w^0 refer to the initial amount (at $t = 0$ by adjusting clocks) of the energy densities of the quintessence and of the Chaplygin gas with respect to the energy density of the van der Waals fluid, respectively and a is a dimensionless cosmic scale factor. The Chaplygin gas could interpolate a matter dominated Universe (pressure-less fluid or dust) where $\rho_c \propto 1/a^3$ when $\psi/a^6 \gg 1$ and a cosmological constant dominated Universe where $\rho_c = -p_c$ when $\psi/a^6 \ll 1$. Hence, the parameter ψ – which is related to the integration constant of the differential equation $(9)_1$ – could be identified with the amount of the pressure-less fluid.

The time evolution of the cosmic scale factor (5) for the mixture of a van der Waals fluid with quintessence becomes

$$\dot{H} + \frac{3}{2}\left\{ H^2 + \frac{8w_w[H^2 - \rho_X^0/(\rho_w^0 a^{3(w_X+1)})]}{3 - [H^2 - \rho_X^0/(\rho_w^0 a^{3(w_X+1)})]} \right.$$
$$-3\left[H^2 - \frac{\rho_X^0}{\rho_w^0}\left(\frac{1}{a}\right)^{3(w_X+1)}\right]^2$$
$$\left. +w_X\frac{\rho_X^0}{\rho_w^0}\left(\frac{1}{a}\right)^{3(w_X+1)} + \varpi \right\} = 0. \quad (13)$$

Equation (13) together with an equation for the non-equilibrium pressure lead to a complete determination of the time evolution of the cosmic scale factor for a mixture of a van der Waals fluid with quintessence. Within the framework of extended (causal or second-order) thermodynamic theory the evolution equation for the non-equilibrium pressure in a linearized theory reads [9]

$$\varpi + \tau\dot{\varpi} = -3\eta H \quad \text{or} \quad \varpi + \alpha\dot{\varpi} = -3\alpha H^3, \quad (14)$$

by assuming that the coefficient of bulk viscosity η and the characteristic time τ are related to the energy density of the mixture $\rho = \rho_w + \rho_d$ (here $\rho_d = \rho_X$) by $\eta = \alpha\rho$ with $\tau = \eta/\rho$ where α is a constant (see e.g. [6]).

Once the cosmic scale factor is determined as a function of time, the energy density of the quintessence can be calculated from $(12)_1$, while the energy density of the van der Waals fluid is obtained from

$$\frac{\rho_w}{\rho_w^0} = \left[H^2 - \frac{\rho_X^0}{\rho_w^0}\left(\frac{1}{a}\right)^{3(w_X+1)}\right]. \quad (15)$$

For the mixture of a van der Waals fluid with a Chaplygin gas instead of (13) we have

$$\dot{H} + \frac{3}{2}\left\{ H^2 + \frac{8w_w[H^2 - \rho_c^0\sqrt{1+\psi/a^6}/(\rho_w^0\sqrt{1+\psi})]}{3 - [H^2 - \rho_c^0\sqrt{1+\psi/a^6}/(\rho_w^0\sqrt{1+\psi})]} \right.$$
$$-3\left[H^2 - \frac{\rho_c^0}{\rho_w^0}\frac{1}{\sqrt{1+\psi}}\sqrt{1+\frac{\psi}{a^6}}\right]^2$$

$$-\frac{\rho_c^0}{\rho_w^0}\frac{1}{\sqrt{1+\psi}\sqrt{1+\psi/a^6}}+\varpi\Bigg\}=0. \qquad (16)$$

From the system of differential equations (14) and (16) it is possible to determine the time evolution of the cosmic scale factor for a mixture of a van der Waals fluid with a Chaplygin gas. Moreover, the energy density of the Chaplygin gas follows from $(12)_2$ and the energy density of the van der Waals fluid can be determined from

$$\frac{\rho_w}{\rho_w^0}=\left[H^2-\frac{\rho_c^0}{\rho_w^0}\frac{1}{\sqrt{1+\psi}}\sqrt{1+\frac{\psi}{a^6}}\right]. \qquad (17)$$

The system of differential equations (13) and (14) for the mixture of a van der Waals fluid and quintessence and the corresponding one (16) and (14) for the mixture of a van der Waals fluid and a Chaplygin gas have been solved by considering the initial conditions: $a(0)=1$ for the cosmic scale factor, $H(0)=1$ for the Hubble parameter and $\dot{\varpi}(0)=0$ for the non-equilibrium pressure. In order to have a complete determination of the time evolution of the cosmic scale factor and of the energy densities there still remains much freedom, since each system of differential equations does depend on four parameters, namely: w_w, ρ_X^0/ρ_w^0, w_X, and α for the mixture with the quintessence and w_w, ρ_c^0/ρ_w^0, ψ, and α for the mixture with the Chaplygin gas. These parameters can be interpreted as follows: (a) w_w is connected with the inflaton, since the increase of w_w leads to a less pronounced inflationary period (see previous section); (b) ρ_X^0/ρ_w^0 and ρ_c^0/ρ_w^0 are related to the initial amount of the energy density of the quintessence and of the Chaplygin gas with respect to the energy density of the van der Waals fluid; (c) w_X takes into account the strength of the negative pressure of the quintessence; (d) ψ – as was previously commented – is related to the amount of the pressure-less fluid in the Chaplygin equation of state and (e) α is connected with the importance of the irreversible processes in the evolution of the Universe.

In the figures 3 through 5 we have chosen $w_w=0.6$, $\rho_X^0/\rho_w^0=\rho_c^0/\rho_w^0=0.03$, $w_X=-0.9$, $\psi=3$ and $\alpha=0.13$ and we proceed to discuss the results.

In the figure 3 it is plotted the "total" pressure as function of the time t for a Universe considered as a: (a) mixture of a van der Waals fluid with quintessence ($p_w+p_X+\varpi$ – straight line); (b) mixture of a van der Waals fluid with a Chaplygin gas ($p_w+p_c+\varpi$ – dashed line) and (c) one-component van der Waals fluid ($p_w+\varpi$ – dotted line). In figure 4 it was adopted the same convention for the lines in the plots of the acceleration field $\ddot{a}$ as a function of time t. We infer from these figures that for the two models of the Universe in which the dark energy is present there exist: (a) an inflationary period with an exponential growth of the acceleration up to a maximum value where the "total" pressure remains with a constant negative value; (b) an interval where the acceleration decreases from its maximum positive value to its maximum negative value corresponding to a growth of the "total"
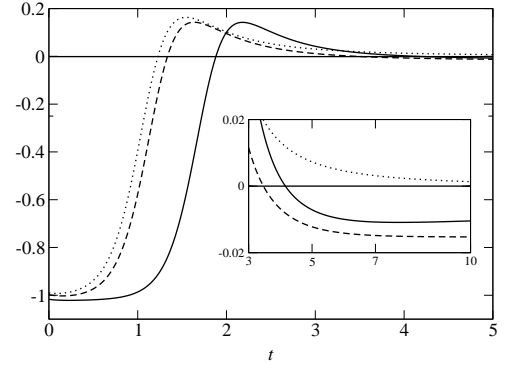

FIG. 3: "Total" pressure vs time for a mixture with quintessence (straight line), for a mixture with Chaplygin gas (dashed line) and for the one-component van der Waals fluid (dotted line).
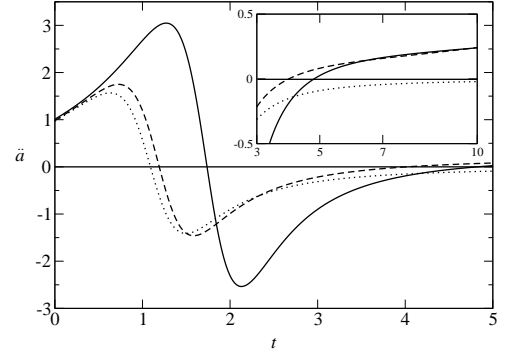

FIG. 4: Acceleration vs time for a mixture with quintessence (straight line), for a mixture with Chaplygin gas (dashed line) and for the one-component van der Waals fluid (dotted line).

pressure from its maximum negative value up to a maximum positive value, and (c) a period where a growth of the acceleration takes place from its maximum negative value to a positive value and where the "total" pressure decays from its maximum positive value to a negative value. Almost the same conclusions can be drawn out here for the Universe modeled by the one-component van der Waals fluid, the difference being the lack of a negative "total" pressure and the corresponding accelerated epoch for the present period of the Universe, which is connected with the absence of the dark energy density. Hence, the mixtures of a van der Waals fluid with quintessence and with a Chaplygin gas can model the three periods of the Universe concerning its acceleration, beginning with an inflationary accelerated period, passing through a past decelerated epoch and leading back to a present accelerated phase, while the one-component van der Waals fluid can model only the two first periods, without an epoch of present acceleration.

The energy densities of the van der Waals fluid ρ_w, of quintessence ρ_X and Chaplygin gas ρ_c together with the pressures of the quintessence p_X and of the Chaplygin
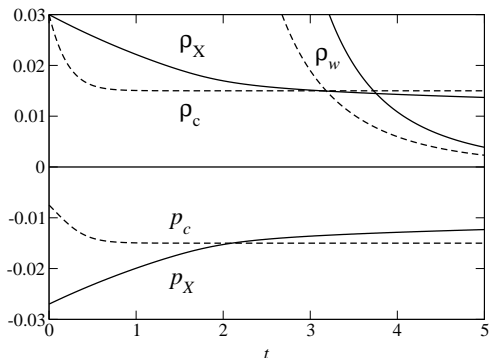
FIG. 5: Energy densities and pressures vs time for a mixture with quintessence (straight lines) and for a mixture with Chaplygin gas (dashed lines).

gas p_c are plotted in figure 5 as functions of time t. As in the figures 3 and 4 the straight lines refer to a mixture of the van der Waals fluid with the quintessence while the dashed lines correspond to a mixture with the Chaplygin gas. We conclude from figure 5 that the energy density of the van der Waals fluid decays more rapidly than the energy densities of the quintessence and of the Chaplygin gas. This fact indicates that these two last energy densities outweigh the first one at later times during the Universe evolution, even if we consider a small amount of the energy density of the quintessence or of the Chaplygin gas with respect to the van der Waals fluid at the beginning (by adjusting clocks). We infer also from this figure that: (a) the energy density of the Chaplygin gas tends to a constant value at early times than the energy density of the quintessence does, and (b) the pressure of the Chaplygin gas at later times is smaller than the pressure of the quintessence. These two observations are connected with the fact that the present acceleration of the mixture with

the Chaplygin gas begins at early times than the corresponding one for the mixture with the quintessence (see also figure 4).

Now we shall comment on the behavior of the solutions when the parameters for the mixture of the van der Waals fluid with the quintessence (w_w, ρ_X^0/ρ_w^0, w_X, α) and for its mixture with the Chaplygin gas (w_w, ρ_c^0/ρ_w^0, ψ, α) are changed. The period of past deceleration begins at earlier times while the period of present acceleration begins at later times when one of the following parameters changes: (a) the amount of the initial energy density of quintessence with respect to the van der Waals fluid ρ_X^0/ρ_w^0 decreases as well as the corresponding one for the Chaplygin gas ρ_c^0/ρ_w^0, (b) the strength of the negative pressure of the quintessence w_X reduces, and (c) the amount of the pressure-less fluid ψ in the Chaplygin equation of state increases. Hence, all these parameters are connected with a more pronounced predominance of the matter field in the decelerating phase of the Universe. The periods of past deceleration and the period of present acceleration begins at earlier times when w_w increases or α decreases. This can be understood by recognizing that both parameters have influence on the early inflationary period, since the increase of w_w leads to a less pronounced inflationary period while the decrease of α implies that at the beginning the non-equilibrium pressure has a less pronounced negative value.

As a final comment let us write the equation of state of the Chaplygin gas as $p_c = w_c \rho_c$. From figure 5 one can infer that for the times where the Chaplygin gas prevails over the van der Waals fluid we have that $-1 \leq w_c < -0.8$. This last result together with the value $w_X = -0.9$, adopted for the quintessence, are in agreement with the observational constraints for w_c and w_X presented by some authors (see e.g. the works [8]).

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